

IIT JEE 2025, Physics Problems

(Quick Learning versus In-depth Learning)

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Quick learning is an illusion. Imitating something is not learning. Quick and learning, the two are opposite attributes. Good result comes only from in-depth learning; quick learning is generally not of much use, it eventually turns out to be wastage of time and resources. In order to learn something, you need to study in depth the concepts involved and solve a variety of problems, which normally takes time. Learning pace depends on individual. There may be quick learners (who are able to learn a subject quickly and deeply); this attribute sets them apart compared to others. Quick learning inevitably leads to poor performance, success comes only from in-depth learning.

Question 1. Consider a star of mass m_2 kg revolving in a circular orbit around star of mass m_1 kg with $m_1 \gg m_2$. The heavier star slowly acquires mass from the lighter star at a constant rate of γ kg/s. In this transfer process, there is no other loss of mass. If the separation between the centers of the stars is r , then its relative rate of change $\frac{1}{r} \frac{dr}{dt}$ (in s^{-1}) is given by:

(A) $-\frac{3\gamma}{2m_2}$ (B) $-\frac{2\gamma}{m_2}$ (C) $-\frac{2\gamma}{m_1}$ (D) $-\frac{3\gamma}{2m_1}$

JEE Advanced 2025, 3 Marks (Bonus)

This problem has many problems. The problem could have been worded better. In fact, conceived better. Here is the deconstruction.

1. Consider a star of mass m_2 kg revolving in a circular orbit around another star of mass m_1 kg with $m_1 \gg m_2$.

Is this the initial arrangement (at $t=0$), or the circular orbit motion holds true for all times? The most sensible answer is this is the arrangement at the moment when you started

looking at the system. Customarily we need an initial arrangement, then we analyze a process (applying conditions and restrictions) to arrive at something.

2. The heavier star slowly acquires mass from the lighter star at a constant rate of γ kg/s. Here, what is meant by ‘slowly acquires’? Should one read it as ‘starts acquiring’ or as ‘has been acquiring’? ‘Starts acquiring’ will occur a billion years later. On the other hand, ‘has been acquiring’ demands an initial configuration at some point of time in deep past, and in that case we have a ‘snake’ of matter connecting the stars. Moreover, in that case how come the lighter star is still able to maintain a *circular orbit*.

3. In this transfer process, there is no other loss of mass.

Why the word *other* in this sentence? It has been clearly stated in the question that the heavier star gains what the lighter star loses. The question doesn’t specify any way (or some ways) of loss of mass, which students were supposed to account for. There is no loss of mass at all. Then why the phrase “no other loss of mass”?

(Considerations of fusion reactions, radiation, ‘star wind’ (like solar wind of our Sun) etc. that involve loss of mass will push the problem in the domain of cosmology. Out of syllabus.)

4. Does the lighter star pump matter out of it or the matter just loses free from it? If it pumps, then the story is fairly complicated;

the circular orbit assumption also breaks down. On the other hand, if matter simply looses free from it, the dissociated matter will have a space walk! The question in that case is this: Why should the dissociated matter go toward the heavier star? It can continue moving in the same circular orbit as it possesses the required orbital speed.

5. We have to look at the dynamical equilibrium of the orbiting star, and also apply the concept of conservation of angular momentum to “a system”. When we do that, what is the mass of our little star and that of the boss? $(m_2 - \gamma t)$ and $(m_1 + \gamma t)$ or $m_2 = m_{02} - \mu t$ and $m_1 = m_{01} + \mu t$, where m_{02} and m_{01} are the masses at the initial instant, or m_2 and m_1 as such?

Does the angular momentum of the lighter star about the heavier star remain constant? Force on mass m_2 by mass m_1 is aimed at m_1 , this force does not produce any torque about the centre of the orbit, but what about the forces on the lighter star by Δm 's that have wandered off?

A word of caution: Don't even think of applying *energy principles* in any way. While reading and attempting this problem, the word *linear momentum* should not even come to your mind. It's a crime. When you read this problem did the terms like *pure rolling* or *melting point of ice* or *Fraunhofer diffraction* come to your mind? Of course, not. You are not a nut. You are smart. Then why should the term linear momentum come to your mind? You know what is relevant, what is not.

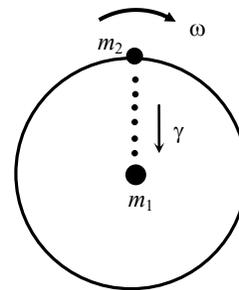
6. In the last, an emotional issue. After all, a star is a star; we must show some respect to it. Writing “Consider a star of mass m_2 kg...” is puerile; ‘*a star mass m_2* ’ is good enough, or ‘*a star of mass m_2 (in kg)*’; we do divide m kg potatoes among n persons, notwithstanding.

In a nutshell, one can't say what exactly is happening and what tools to apply to analyze

it. The true picture is extremely complicated. The paper setter must have thought of all these complications, he might have thought far more; he may be an authority on cosmology, who knows. It seems what he did is this: he entered the mind of a XII class student and created a very simple model.

A star is orbiting (in a circular path) a heavier star, matter comes out of the lighter star and is received by the heavier one, all Δm 's somehow lie on the line joining the stars *all the time*, the two stars and all Δm 's beautifully arranged on a rotating line, Δm 's inching closer toward the heavier star, and in this process (in view of the professor who framed this problem) the lighter star is also being pulled in. (We will see that the lighter star, in fact, recedes.)

What a beautiful model!



Now we come to the formulation. The dynamical equilibrium equation and the angular momentum of the smaller star may be written as

$$\frac{G(m_1 + \gamma t)(m_2 - \gamma t)}{r^2} = (m_2 - \gamma t)\omega^2 r$$

and $L = (m_2 - \gamma t)\omega r^2$.

So many approximations and yet such ugly equations! What nonsense!!

Let's go for simple, beautiful equations

$$\frac{Gm_1 m_2}{r^2} = m_2 \omega^2 r \quad \dots(i)$$

and $L = m_2 \omega r^2 \quad \dots(ii)$

(Even writing $\frac{Gm_1 m_2}{r^2} = m_2 \left(\omega^2 - \frac{d^2 r}{dt^2} \right)$ does

not make any sense, given the simplifying assumptions. Nothing perceptible happens

here in small time intervals - a minute or an hour or a day or a week or a year. It's game of billions of years.)

After eliminating ω from Eqs. (i) and (ii), we rearrange the terms a little and write the resulting equation as

$$m_1 m_2^2 r = \frac{L^2}{G} = \text{constant} \quad (\text{You know why}).$$

After some simple mathematics* (logarithm and differentials) we get

$$\frac{1}{m_1} \frac{dm_1}{dt} + \frac{2}{m_2} \frac{dm_2}{dt} + \frac{1}{r} \frac{dr}{dt} = 0$$

or
$$\frac{1}{m_1} (\gamma) + \frac{2}{m_2} (-\gamma) + \frac{1}{r} \frac{dr}{dt} = 0.$$

(Note: $\frac{dm_1}{dt} = \gamma, \quad \frac{dm_2}{dt} = -\gamma.$)

Since $m_1 \ll m_2$, we can ignore the term $\frac{\gamma}{m_1}$ in

the above equation, and finally obtain

$$\frac{1}{r} \frac{dr}{dt} = \frac{2\gamma}{m_2}.$$

That's it. Don't insert a minus sign for the heck of arriving at one of the given choices.

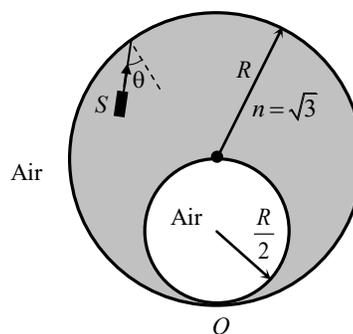
When someone starts robbing you, you run away from him, you do not rush to his door. Our little star is as intelligent as you are! It recedes from the heavier star.

*This mathematics is very disturbing. We never do it that way. One can challenge it - when you wrote Eqs. (i) and (ii), you tacitly assumed m_1 and m_2 to be constant, then how can you treat them as variable next very moment? Our crime is indefensible. Typically, we should look at the system at time t , taking masses as m_1 and m_2 , and then at time $t + \Delta t$ taking masses as $m_2 - \Delta m$ and $m_1 + \Delta m$. Invoking the basic laws we get an equation to play with. All that just for 3 marks? Good heavens!

Question 2. A solid glass sphere of refractive index $n = \sqrt{3}$ and radius R contains a spherical

air cavity of radius $\frac{R}{2}$, as shown in the figure.

A very thin glass layer is present at the point O so that the air cavity (refractive index $n=1$) remains inside the glass sphere. An unpolarized, unidirectional and monochromatic light source S emits a light ray from a point inside the glass sphere towards the periphery of the glass sphere. If the light is reflected from the point O and is fully polarized, then the angle of incidence at the inner surface of the glass sphere θ . The value of $\sin\theta$ is.....



JEE Advanced 2025, 4 Marks

First, a few observations:

1. A solid glass sphere of refractive index $n = \sqrt{3}$ and radius R ...

Here, is the refractive index $n = \sqrt{3}$ characteristic to the glass or to the sphere? The radius R is characteristic to the sphere, undoubtedly. Refractive indices are referred to a medium (material), not to a body. If $n = \sqrt{3}$ is the refractive index of the glass, then what is wrong in taking R as the radius of the glass? How to read this sentence?

2. ...and monochromatic light source S emits a light ray from...

Do light sources emit light or light rays? Rays feature in while representing the propagation of light. Light sources emit light, waves, wave trains, photons... Do they emit rays also? We do say a ray emanated from a light source!

3. If the light is reflected from the point O and is fully polarized, then the angle of incidence at the inner surface of the glass sphere is θ .

Does the reflection at the point O happen first, the incidence at the inner surface of the glass sphere later? What is *if* and what is *then* in this sentence? Whether ' θ ' is such that light is "reflected from the point O and..." or if the light is reflected from the point O and is fully polarized, *then* the angle of incidence at the inner surface of the sphere is θ ?" What precedes what? Moreover, does the light get polarized at the point O , or was it already polarized (before hitting the point O)?

"... light is reflected from the point O and is fully polarized..." is very different from "... light is reflected from the point O and gets polarized..." Did polarization occur at the point O or a year ago?

Is it "..., *then* the angle of incidence at the inner surface of the glass sphere is θ " or "..., *when* the angle of incidence at the inner surface of the glass sphere is θ "?

An already polarized light falling at the point O can also be a possibility; polarization occurring at O is certainly a case.

(Official JEE site also gives two answer- 0.75 and 0.50.)

4. What were the students supposed to do? Explore different possibilities, or once they arrived at an answer with a suitable ray diagram, then become very sure that that was the answer? Somewhat similar to the Uniqueness Theorems of electrostatics, where if you could guess a solution, that would be the only solution.

Why should students explore different possibilities? There is no hint in the question to this effect. What was asked is this:

5. The value of $\sin \theta$ is...

If the professor who framed this problem was aware that there were two possibilities, he would have better asked-

The value(s) of $\sin \theta$ is/are...

It doesn't make much sense to write the detailed solution. Here is the sketchy one.

(i) When polarization occurs at the point O

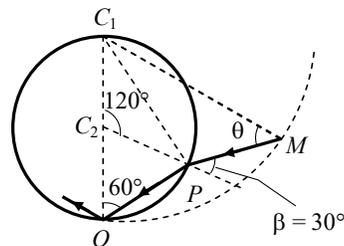
Using the Brewster angle concept we can show that the light must be incident at the point O at a angle given by $\tan \theta_b = \sqrt{3}$ or $\theta_b = 60^\circ$.

Figure below shows the required details.

Apply Snell's law at point P -

$$\sqrt{3} \sin \beta = 1 \times \sin 60^\circ$$

which gives $\beta = 30^\circ$.

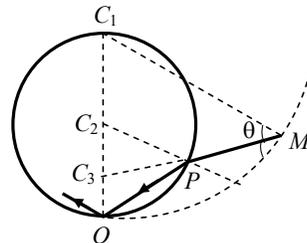


We have $C_1P = 2 \times \frac{R}{2} \sin 60^\circ = \frac{R\sqrt{3}}{2}$. We can show that $\angle C_1PM = 120^\circ$. Thus,

$$\frac{\left(\frac{R\sqrt{3}}{2}\right)}{\sin \theta} = \frac{R}{\sin 120^\circ},$$

which gives $\sin \theta = \frac{3}{4} = 0.75$.

Alternatively, just extend the line MP . It will meet the line C_2O at its mid point, and the angle C_1C_3M will be 90° , see figure below.

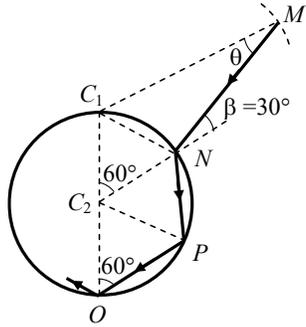


Now

$$\sin \theta = \frac{C_1C_3}{C_1M} = \frac{\frac{R}{2} + \frac{R}{4}}{R} = \frac{3}{4} = 0.75.$$

(ii) Polarized light is incident at the point O

Polarization occurs at point P , shown in the figure below. Polarized light falls at O .



You can show $C_1N = \frac{R}{2}$, and $\angle C_1NM = 90^\circ$.

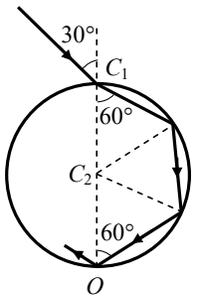
In ΔC_1NM

$$\sin \theta = \frac{C_1N}{C_1M} = \frac{\left(\frac{R}{2}\right)}{R}$$

or $\sin \theta = \frac{1}{2} = 0.50$.

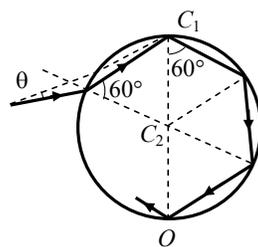
Are there other possibilities also? Do we need more exploration? Let us see.

(a)



Straightaway rejected.

(b)



You can show that $\theta < \theta_c$.
Ruled out.

(c)

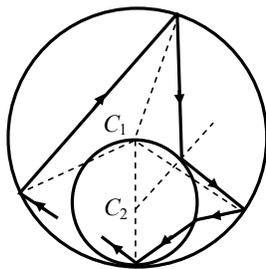


Fig...

(A Civil Engineering student at IIT BHU, Varanasi, Sutirtha Biswas did some analysis

and concluded that this case is not possible. In his analysis Sutirtha invoked some concepts of coordinate geometry in the most ingenious way. This approach reduces the burden of tons of calculations. A few faculty members are exploring if polarization could have occurred at the surface of the glass sphere itself. I wish them success.)

Question 3. A person sitting inside an elevator performs a weighing experiment with an object of mass 50 kg. Suppose that the variation of the height y (in m) of the elevator, from the ground, with time t (in s) is given by

$$y = 8 \left[1 + \sin \left(\frac{2\pi t}{T} \right) \right], \text{ where } T = 40\pi \text{ s. Taking}$$

acceleration due to gravity, $g = 10 \text{ m/s}^2$, the maximum variation of the object's weight (in N) as observed in the experiment is...

JEE Advanced 2025, 4 Marks

Truly speaking, the person sitting in the elevator is not doing anything, except enjoying the ride.

Let us imagine what experiment he could perform in order to determine what he wanted to. But some quick calculations first -

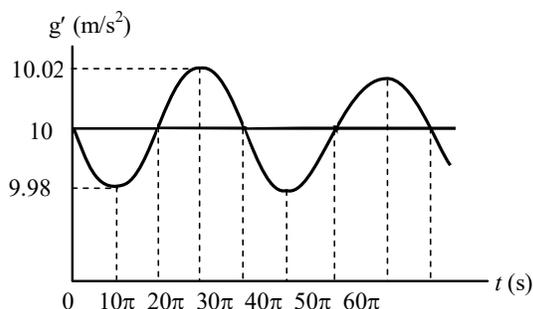
$$T = 40\pi \text{ s} = 126 \text{ s,}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{40\pi} = \frac{1}{20} \text{ rad/s,}$$

$$v_{\max} = \omega A = \frac{1}{20} \times 8 = 0.4 \text{ m/s,}$$

$$a_{\max} = \omega^2 A = \left(\frac{1}{20} \right)^2 \times 8 = 0.2 \text{ m/s}^2.$$

The person is in a region where the acceleration due to gravity g' varies sinusoidally from $g'_{\min} = 10 - 0.02 = 9.98 \text{ m/s}^2$ to $g'_{\max} = 10 + 0.02 = 10.02 \text{ m/s}^2$.



The person needs to measure g'_{\min} and g'_{\max} . What experiments could he perform for this in the elevator? May be the simple experiments we perform on the ground to measure g - dropping a body from some height and recording the time of fall, measuring the time period of a simple pendulum, and others.

A pendulum inside another 'pendulum' (an oscillating system for those who do not like poetry) is a very fancy device. Our experimenter sticks to basics and decides to record the time of fall. Besides, he is performing a weighing experiment with an object of mass 50 kg. He raises the 50-kg object to a height (say 2 m), drops it, and records the time of its fall. He repeats this experiment many a time. (Weight lifting in a lift!)

The notebook of the experimenter shows a range of values for the time of fall. He calculates the corresponding values of g' ,

from $2 = \frac{1}{2} g' t^2$. He needs g'_{\min} and g'_{\max} .

(We on the ground know that his notebook entries may have a certain t_{\min} and t_{\max} given by

$$2 \pm x = \pm 0.4t + 5t^2.$$

We expect x to be very small, pay attention to the values $v_{\max} = 0.4 \text{ m/s}$ and $T = 126 \text{ s}$. For the time being we ignore x , to get

$$2 = \pm 0.4t + 5t^2$$

which gives $t_{\min} = 0.59 \text{ s}$ and $t_{\max} = 0.67 \text{ s}$, provided the person drops the object exactly at the instant when the elevator was passing through its mean position. Given the complexity of the arrangement, can he

determine t_{\min} and t_{\max} with certainty? Compare the values of t_{\min} and t_{\max} with T (the time period of elevator's motion).

How many times he needs to repeat this experiment to arrive at the acceptable values of g'_{\min} and g'_{\max} . Body building can't go on indefinitely!

Then? What next?

He borrows a weighing machine from us, the machine designed and calibrated and manufactured by us on the ground. He puts the 50-kg object on the machine, and looks at the indicator for 10-15 minutes continuously. He needs to record the extreme values many a time. Then he tells us -

the maximum variation of the objects weight is 2 N, as if he were reporting the variation of "colour and texture" of something, not the variation in its numerical value, whereas we lesser mortals on the ground calculated the maximum variation in object's weight as

$$\left[50 \times \left(10 + \frac{1}{50} \right) \right] - 50 \text{ or } 50 - \left[50 \times \left(10 - \frac{1}{50} \right) \right],$$

which is 1 N.

Since he was the one who was 'performing the experiment', we were unnecessarily doing physics and dirty calculations, we have to accept his result.

After all this nonsense, I come to something meaningful. A rank in JEE Advanced matters. Ranks depend on the score. Score depends on how accurately you solve a problem within the time frame. After reading this question a student should write only this:

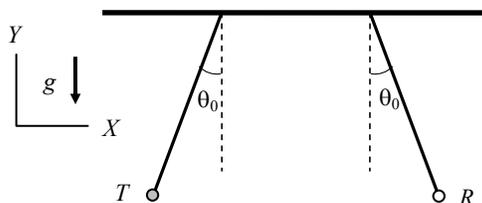
$$2 \times m\omega^2 A$$

$$2 \times 40 \times \left(\frac{2\pi}{40\pi} \right)^2 \times 8 = 2 \text{ N.}$$

And, I am sure, most of the serious students would have done exactly this.

Question 4. An audio transmitter (T) and a receiver (R) are hung vertically from two identical massless string of length 8 m with their pivots well separated along the X axis.

They are pulled from the equilibrium position in opposite directions along X axis by a small angular amplitude $\theta_0 = \cos^{-1}(0.9)$ and released simultaneously. If the natural frequency of the transmitter is 660 Hz and the speed of sound in air 330 m/s, the maximum variation in the frequency (in Hz) as measured by the receiver (Take the acceleration due to gravity $g = 10 \text{ m/s}^2$) is...



JEE Advanced 2025, 4 Marks

First thing first. What a serious, hardworking student did (or would have done) to answer this question in exam? Just this:

$$v = \sqrt{2 \times 10 \times 8(1 - 0.9)} = 4 \text{ m/s}^{**}$$

$$660 \times \frac{334}{326} - 660 \times \frac{326}{334} = \dots = \dots = 32.$$

One fills the missing steps one's own way.

This was a 4- mark, 4-minute problem.

*If your concepts and logic are in place, physics is a very easy subject. In exams like JEE Advanced, you have to be both quick and lazy. Quick in terms of thinking about the solution steps and lazy in terms of solution steps you write. Ideally, by the time you finish reading a problem, your mind should roughly construct the steps to solve it. By the second reading, if required, you must be ready to write what is absolutely essential.

** As an example, in this problem you should not waste your time in calculating T, ω, A as they are inconsequential in solving it. Even if you write $\frac{1}{2}mv^2 = mg(l - l \cos \theta_0)$, then $v = \sqrt{2gl(1 - \cos \theta_0)}$, and then you insert numbers, you lose precious time. You must have done this a thousand times in the last two years. You must have calculated $l(1 - \cos \theta)$ in

hundreds of problems. (If you think that $v^2 = u^2 + 2aS$ will give you the required v , God save you!)

*** Mostly, as a matter of habit or training, students first write a formula, then insert numbers. They do this even in the case of simplest of the equations, $v = u + at$ or $m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$. I fail to fathom why. Formula is in your head, why don't you simply write, for example, $v = 4 + (-3) \times 7$ or $2 \times (-2) + 4v_2 = 2 \times 5 + 4(-1)$ straightaway?

Let us do some physics now.

1. ...with their pivots well separated along the X axis.

Why the pivots are to be well separated? Just in order to avoid a collision between the transmitter and the receiver? Or there is something more fundamental to it? The way they are set in motion, they will dance in synchronism. If the pivots are not well separated, will the emission and detection of sound waves be affected in some way? How close can the transmitter come to the receiver? We do whisper in someone's ear. The wavelength of the sound wave in this example is $\frac{330}{660} = 0.5 \text{ m}$. Does this number somehow decides the separation between the pivots?

How is a XII class student supposed to use this information in solving this problem? Should he assume a specific value for the separation between the pivots (500 m or 300 m or 100 m), then proceed, or should he ignore this information and move ahead?

If a student assumes this separation to be 100 m, his task becomes arduous. He will have to do lots of cumbersome calculations. (Read page 75 of Textbook of Physics Volume 2, by Neelam, Hedge and Kalyan.) Many students may have stuck in decoding this information and lost their composure.

2. They are pulled from the equilibrium position in opposite directions along X axis by...

Under the given condition the oscillating bodies will go off the X axis by approx 1 m (80 cm to be precise). Motion occurs along an arc that subtends an angle of 52° at the point of suspension. There seems to be a purpose in choosing the values $l=8$ m and $\theta_0 = \cos^{-1}(0.9)$. More focus on concepts than calculations; playing with simple numbers. But there are other choices that fit in the scheme perfectly. They could have gone, for example, for a string of length 20 m without annoying Foucault, and $\theta_0 = \cos^{-1}(0.99)$. Simplification steps remain the same, one gets a good numerical value for speed, and bodies practically lie on the X axis all the time.

A more puzzling question is why at all an X axis is required; and bodies to be displaced along this axis. Let them swing. After all what matters is their speed at the mid points of the their paths; whether they swing along an arc or move on a straight line is immaterial.

3. They are pulled... by a small angular amplitude $\theta_0 = \cos^{-1}(0.9)$ and...

Or by a small angle θ_0 ? Angle θ_0 becomes the angular amplitude once they start oscillating.

4. Student were asked to calculate the maximum variation in the frequency as measured by the receiver. Unless the separation between the pivots is given specifically and that number enters in calculations, this is a simple, straightforward problem.

The maximum variation in the frequency received by the receiver is simply equal to the maximum frequency it measures minus the minimum frequency it measures. In the given situation f_{\max} and f_{\min} will have unique values, not a range, and $f_{\max} - f_{\min}$ will be one

(definite) number. These values correspond to the position and velocity of the transmitter and the receiver as shown in the figure below.

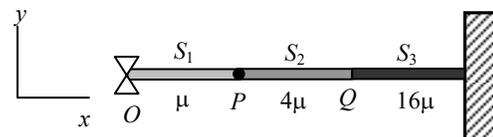


(This statement is based on the assumption that there is no time lag between emission and detection of the sound wave.)

But in the official answer it is a range (26 to 33). Is the receiver whimsical; it receiver what it wants to? Did it skip a few cycles, stayed back where it was? Or, the emitter played a game with the receiver?

Also, the maximum variation in the frequency sought in the problem can't be equal to $f_{\max}(\text{received}) - f_0(\text{natural frequency of the source})$ or $f_0 - f_{\min}$. The receiver does not know anything about the source, it is not supposed to. *What matters to it is what it receives!*

Question 5. Consider a system of three connected strings, S_1, S_2 and S_3 with uniform linear mass densities μ kg/m, 4μ kg/m and 16μ kg/m, respectively, as shown in the figure. S_1 and S_2 are connected at the point P , whereas S_2 and S_3 are connected at the point Q , and the other end of S_3 is connected to a wall. A wave generator O is connected to the free end of S_1 . The wave from the generator is represented by $y = y_0 \cos(\omega t - kx)$ cm, where y_0, ω and k are constants of appropriate dimensions. Which of the follow statement is are correct:



(A) When the wave reflects from P for the first time, the reflected wave is represented by $y = \alpha_1 y_0 \cos(\omega t + kx + \pi)$ cm, where α_1 is a positive constant.

(B) When the wave transmits through P for the first time, the transmitted wave is represented by $y = \alpha_2 y_0 \cos(\omega t - kx)$ cm, where α_2 is a positive constant.

(C) When the wave reflects from Q for the first time, the reflected wave is represented by $y = \alpha_3 y_0 \cos(\omega t - kx + \pi)$ cm, where α_3 is a positive constant.

(D) When the wave transmits from Q for the first time, the transmitted wave is represented by $y = \alpha_4 y_0 \cos(\omega t - 4kx)$ cm, where α_4 is a positive constant.

JEE Advanced 2025, 4 Marks

(Solution by Kalyan)

It turns out that none of the options are correct, because the origin of coordinate system is not clearly specified.

Options B and C are definitely not correct, (A) and (D) can be conditionally correct, but not simultaneously!

Even the correctness of A and D depends on the choice of origin which is not mentioned in the question.

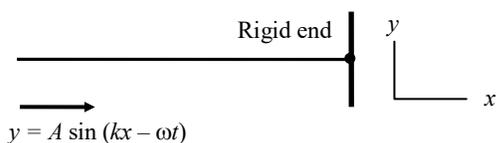
Why is origin important in these kinds of problems?

Let us understand this by considering reflection of wave at rigid end.



We know that when a wave gets reflected at the rigid end, it undergoes a phase change of π . This translates to the fact that a positive displacement reaching the rigid end, at very instant translates to a negative displacement to maintain the net displacement of rigid end as zero. (After all, the rigid end cannot move!)

Assume that the wave $y = A \sin(kx - \omega t)$ is incident rigid end.



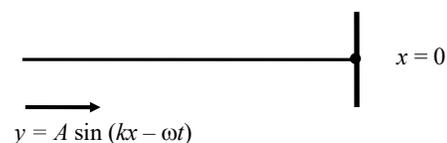
Let's first assume the rigid end to be $x = 0$. Hence the displacement reaching the rigid end at time t is $A \sin(k(0) - \omega t) = -A \sin(\omega t)$, so the reflected wave travels leftwards and its equation must contain $kx + \omega t$ term. The equation satisfying these conditions for reflected wave must be $y_r = A \sin(kx + \omega t)$.

Thus, we always write the reflected wave based on the boundary conditions!

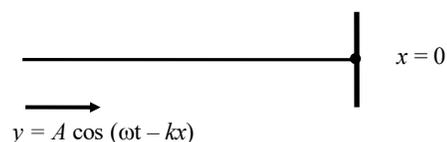
A word of caution: A majority would replace $kx - \omega t$ or $\omega t - kx$ with $kx + \omega t$ and add an extra π to phase to get reflected wave! This may not always work.

It only works if you have $\omega t - kx$ with boundary at origin, irrespective of sine or cosine function.

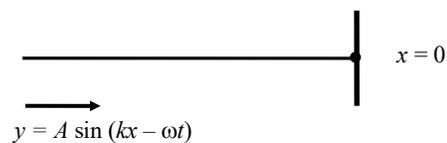
For instance look at below cases:



$y_r = A \sin(\omega t + kx - \pi)$, works!

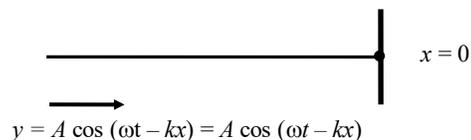


$y_r = A \cos(\omega t + kx + \pi)$, works!



$y_r = A \sin(kx + \omega t + \pi)$ is wrong

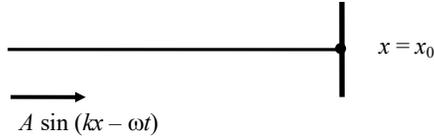
It's better to first write $y = A \sin(kx - \omega t) = -A \sin(\omega t - kx)$. Now $y_r = -A \sin(\omega t + kx + \pi)$, works!



So $y_r = A \cos(\omega t + kx + \pi)$ works!

However, instead of doing this kind of quick fix, it is always better to write proper boundary conditions and then proceed.

What if the boundary is not at $x=0$? This is even more complex.



Let the reflected wave be $A \sin(kx + \omega t + \phi)$.

Then we must have

$$A \sin(kx - \omega t) + A \sin(kx + \omega t + \phi) = 0.$$

$$\text{or } 2A \sin(kx_0 + \phi/2) \cos(\omega t + \phi/2) = 0.$$

$$\text{or } \sin(kx_0 + \phi/2) = 0.$$

$$\Rightarrow \phi = -2kx_0.$$

So, the reflected wave is $A \sin(kx + \omega t - 2kx_0)$.

Alternatively, we can arrive at the above equation by first writing incident wave as

$$\begin{aligned} y_i &= A \sin(kx - \omega t) \\ &= -A \sin(\omega t - kx) \\ &= -A \sin\{\omega t - k(x - x_0) - kx_0\}. \end{aligned}$$

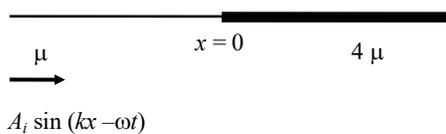
Now we replace $-k(x - x_0)$ with $+k(x - x_0)$ and add an additional π .

$$\begin{aligned} \text{So } y_r &= -A \sin\{\omega t + k(x - x_0) - kx_0 + \pi\} \\ &= A \sin\{\omega t + kx - 2kx_0\}. \end{aligned}$$

Now, let's come to the case where the boundary is not perfectly rigid, but a transition from rarer to denser medium takes place. The wave is traveling in the rarer medium and about to enter the denser one.

In such cases, the reflected wave again undergoes a phase change of π but the amplitude of the reflected wave will be lesser than that of the incident wave. Also the transmitted wave will have lesser amplitude but it will be in phase with the incident wave.

Consider the following scenario:

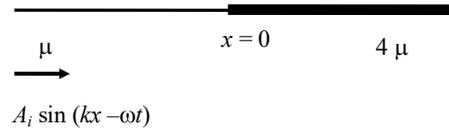


Here, the reflected wave is $A_r \sin(kx + \omega t)$ and transmitted wave is $A_t \sin(2kx - \omega t)$.

Note that the frequency of the wave does not change during reflection or transmission, but the speed and angular wave number change on changing media, i.e., during transmission. Why did we write the transmitted wave as $A_t \sin(2kx - \omega t)$?

Because, at $x = 0$, the phase of transmitted wave is $-\omega t$, which is same as phase of incident wave!

Now, what if the boundary is not $x = 0$, but rather $x = x_0$?



The reflected wave from our previous discussion is $A_r \sin(kx + \omega t - 2kx_0)$, with $A_r < A_i$.

But what about the transmitted wave?

Let's assume it as $A_t \sin(2kx - \omega t - \phi)$, where we must find ϕ using boundary conditions, which are $(kx - \omega t)|_{x=x_0^-} = (2kx - \omega t + \phi)|_{x=x_0^+}$.

[In essence, the phase of incident and transmitted waves are same at boundary.]

Solving, we get $\phi = -kx_0$.

Hence the transmitted wave is

$$A_t \sin(2kx - \omega t - kx_0).$$

Now, with all this background when we look at the JEE problem, we have the following predicament. Where should we take our origin?

If we take origin at O (the point where source is located), then the coordinates of P and Q will be some x_P and x_Q . In that case the reflected wave at P must be

$$-\alpha_1 y_0 \cos(\omega t + kx - 2kx_P).$$

In this situation none of the options in the question are correct!

With origin at O (the source), transmitted wave into the region S_3 will have the equation $\alpha_3 y_0 \cos(\omega t - 4kx + 2kx_Q + kx_P)$. (Do the calculation!)

As a hint, take equation of transmitted wave in S_2 as $\alpha_2 y_0 \cos(\omega t - 2kx + kx_P)$ and proceed to get above result.

What if we take origins at P ? Then (A) is correct but (D) is wrong. However, in the official key (D) is correct. Both options (A) and (D) will be correct only if origin is at Q . However for (A) and (D) to be simultaneously correct, origin must simultaneously be at P and Q , an insanity!